Adaptive Model Order Reduction Techniques for High-Fidelity Multibody Vehicle Models

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1. Background and Motivation

NASA Relevance

- Mobility prediction in soft terrain is essential in extraterrestrial rover missions.
- Past rovers have become stuck in embedding events due to soft sand, losing valuable mission time (e.g. *Spirit*).
- Simulation-based mission planning and decision making are critically important to ensure the success of missions.

Existing Off-Road Mobility Models and Limitations

- Classical terramechanics models are low fidelity and require extensive test data for the model development, which are not always readily available.
- High-fidelity physics-based computational models are accurate but incur a very high computational cost, making them too slow to use in mission planning.
- **Model order reduction (MOR)** techniques seek to simplify high-fidelity models in order to improve their computational efficiency while maintaining their accuracy, but **no studies have been conducted concerning MOR for high-fidelity off-road mobility simulation**.

Fundamental Research Questions

1. Can the advanced MOR technique be a promising approach, eliminating computational bottleneck of high-fidelity off-road mobility simulations while ensuring solution accuracy?
2. How can a reduced model be made adaptive and more robust to changes in vehicle (rover) operating parameters (e.g. soil properties, wheel loads, etc.)?

Objectives

1. **Develop a numerical procedure for a projection-based model order reduction technique for a high-fidelity off-road mobility model** with proper orthogonal decomposition (POD).
2. Apply the adaptive POD technique based on the Grassmann manifold to **account for changes in model parameters as well as operating conditions for practical use**.
3. **Conduct performance evaluation** of the adaptive reduced-order off-road mobility model.

2. Full Order Mobility Model

Model Description

In order to investigate the fundamental capabilities of the adaptive MOR in high-fidelity off-road mobility models, we use the nonlinear tire-soil interaction model previously developed and validated against test data in our lab (Yamashita, 2016). The full order model (FOM) is composed of a nonlinear finite element (FE) tire with a rigid rim rolling over FE soil patches at a constant velocity. As the tire rolls towards the edge of a soil patch, the degrees of freedom (DOF) far behind the tire are "deactivated" and an equal number of DOF ahead of the tire are "activated" through the restart capabilities of the code. This allows for DOF in the soil to be excluded in the computations if they aren't changing. This leads to the following coupled differential-algebraic equations (DAEs) for the dynamic tire-soil soil interaction problem:

$\mathbf{M}_r\ddot{\mathbf{q}}_r + \mathbf{C}_{\mathbf{q}_r}^T\boldsymbol{\lambda} = \mathbf{Q}_r(\mathbf{q}_r, \mathbf{e}_t, \dot{\mathbf{q}}_r, \dot{\mathbf{e}}_t)$	: Rigid bodies	(14 eqs)
$\mathbf{M}_t\ddot{\mathbf{e}}_t + \mathbf{C}_{\mathbf{e}_t}^T\boldsymbol{\lambda} = \mathbf{Q}_t(\mathbf{q}_r, \mathbf{e}_t, \mathbf{e}_s, \dot{\mathbf{q}}_r, \dot{\mathbf{e}}_t, \dot{\mathbf{e}}_s, \boldsymbol{\alpha})$	: Nonlinear FE tires	(12,000 eqs)
$\mathbf{M}_s\ddot{\mathbf{e}}_s = \mathbf{Q}_s(\mathbf{e}_t, \mathbf{e}_s, \dot{\mathbf{e}}_t, \dot{\mathbf{e}}_s)$	: Joint constraints	(8 eqs)
$\mathbf{C}(\mathbf{q}_r, \mathbf{e}_t, t) = \mathbf{0}$	:Deformable FE soil	(2,395 eqs)
		34,417 eqs in total

The preceding 34,417 equations are integrated forward in time at each time step by an implicit time integrator involving an iterative solution process. A large portion of the computational time is spent solving this system of equations, and the larger the number of coordinates, the more computationally expensive it becomes. Use of projection-based model order reduction techniques can potentially allow for decreasing the model dimensionality and speeding up the solution process in the time-domain analysis.

3. Model Order Reduction Techniques

Proper Orthogonal Decomposition (POD) - Background

- POD is a statistical method that decomposes systems with a large number of interdependent variables into a smaller number of completely independent variables (Kerschen, 2005).
- In the computational fluid dynamics (CFD) field, POD has been successfully used to find underlying structures in fluid flows and to reduce computational time (Aubry, 1991), but no studies have been done applying POD to off-road mobility models.

Numerical Procedure for the Model Order Reduction

- The reduced order basis in POD is defined as to minimize the difference between the full and reduced-order solutions over time as in Eq. (1).
- $\boldsymbol{\phi}_k$ is the orthonormal reduced order basis for the *k*-th reduced order component. Therefore, the physical coordinate is approximated as in Eq. (2).
- To find the reduced order basis from the snapshots of the full-order model simulations, the singular value decomposition (SVD) of the snapshot matrix $\mathbf{X}$ is performed and reduced order bases $\boldsymbol{\Phi}$ are obtained as in Eq. (4)
- The reduced order bases are then used to reduce the size of the FE tire and soil model dimensionality in the iterative solution process, thus decreasing the computational time.

$$\{\boldsymbol{\phi}_k\}_{k=1}^{n_\phi} = \underset{\{\boldsymbol{\phi}_k\}_{k=1}^{n_\phi}}{\arg\min} \int_{t_0}^{t_N} \left\| \mathbf{x}(t) - \mathbf{P}\mathbf{x}(t) \right\|^2 dt \quad (1)$$

where $\mathbf{P} = \sum_{k=1}^{n_\phi} \boldsymbol{\phi}_k \boldsymbol{\phi}_k^T$

$$\mathbf{x}(t) = \sum_{k=1}^{n_\phi} \boldsymbol{\phi}_k \xi_k \quad (2)$$

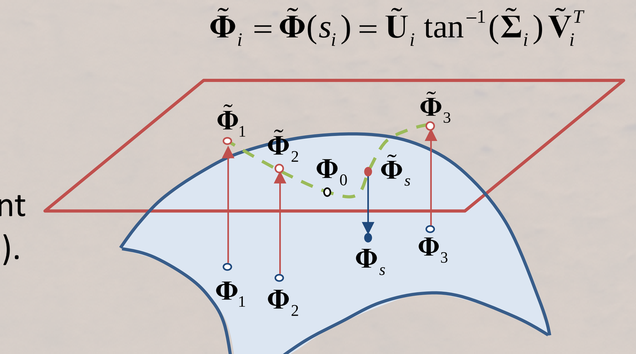
$$\mathbf{X} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T \quad (3)$$

$$\boldsymbol{\Phi} = \{\mathbf{U}_k\}_{k=1}^{n_\phi} \quad (4)$$

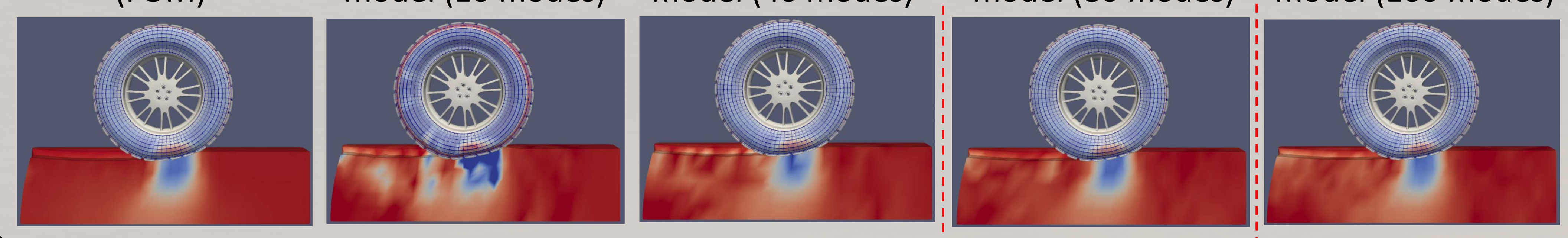
Adaptive POD Modes

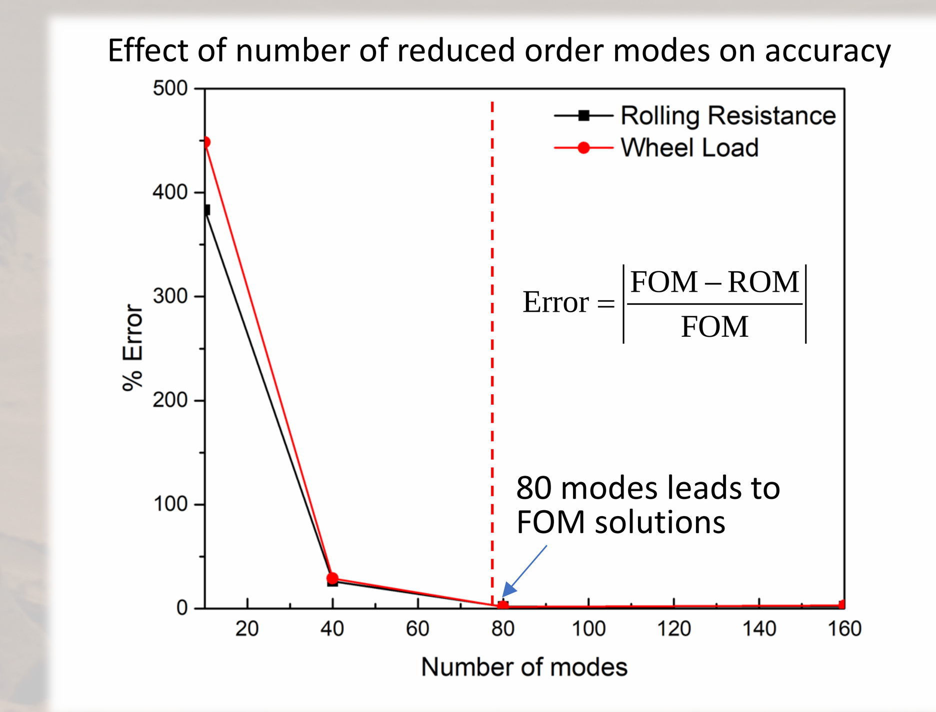
- POD modes for one scenario aren't appropriate for any other scenario, so adaptation of POD modes is required to produce modes for scenarios without FOM simulations.
- Directly interpolating between POD modes is not appropriate because the orthogonal property of POD modes is not ensured if POD modes are directly interpolated.
- As orthogonal bases of the same rank all lie on the same Grassmann manifold, POD bases can be projected onto the tangent space of the manifold at the point of interest and then they are safely interpolated in this space (Amsallem, 2008).
- This mode adaptation approach is applied to off-mobility ROM to account for changes in model parameters as well as operating conditions.

$$\tilde{\boldsymbol{\Phi}}_i = \tilde{\boldsymbol{\Phi}}(s_i) = \tilde{\mathbf{U}}_i \tan^{-1}(\tilde{\boldsymbol{\Sigma}}_i)\, \tilde{\mathbf{V}}_i^T$$



4. Numerical Results

	Full order model (FOM)	Reduced order model (10 modes)	Reduced order model (40 modes)	Reduced order model (80 modes)	Reduced order model (160 modes)
					
Comp. Time	128 min.	38 min.	45 min.	51 min.	56 min.

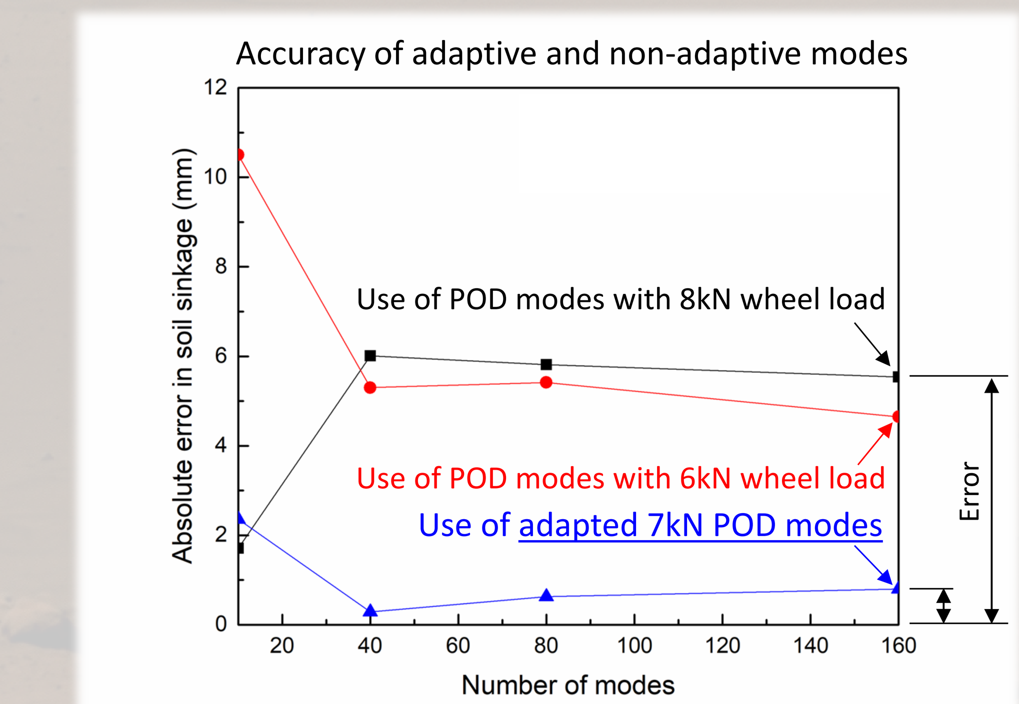


Results of Reduced Order Off-Road Mobility Model

- Snapshots from a FOM with a 6kN wheel load are used to create POD modes for a reduced order model (ROM) with a 6kN wheel load. It is observed from the figure on the left that **the steady-state rolling resistance as well as the wheel load of the ROM agree with those of the FOM if a sufficient number of POD modes are considered**. The solutions of the 80-mode ROM agree with the FOM solutions.
- From the contours above, it is revealed that as the number of modes increases, the stress distribution in the tire and soil approach that of the FOM. The solutions for the 80-mode ROM are sufficiently close to those of the FOM solutions.
- **60% reduction of computational time is achieved with the 80-mode ROM.**

Accuracy of Adaptive MOR with Grassmann Manifold

- POD modes generated from FOMs with 6kN and 8kN wheel loads are used as the sample points for a Grassmann manifold interpolation (i.e., adaptive POD modes) to predict the modes for a 7kN wheel load scenario. The 7kN scenario is then run as a ROM with each of these three sets of POD modes.
- The number of modes was varied until the solutions converged as shown in the figure below. The 7kN FOM was then run to validate the results of the ROMs.
- **The adapted 7kN modes accurately predict the sinkage of the 7kN FOM**, while the 6kN modes and 8kN modes fail to predict the FOM solution. Modes from different scenarios cannot properly reduce other scenarios but the adaptive POD modes function well.

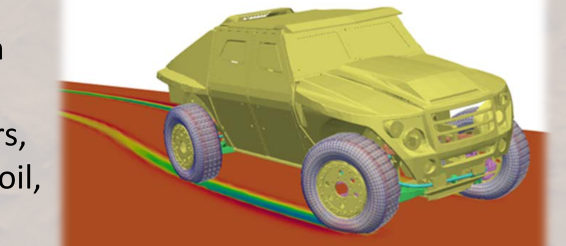


5. Conclusions and Future Work

Summary and Conclusions

In this exploratory study, a numerical procedure for projection-based model order reduction of the high-fidelity off-road mobility model was developed. Significant computational time reductions were demonstrated without large losses of accuracy. It was also demonstrated that the modes generated for one operating parameter are not appropriate for any other operating parameter, necessitating some form of adaptation. To address this problem, a method of interpolation of reduced order bases on the tangent space of the Grassmann manifold was implemented and applied to the reduced order off-road mobility model. With this approach, it was shown that adapted POD modes allow for predicting tire-soil interaction behavior, for which the high-fidelity model had not previously been run. **This is the first time that the adaptive MOR techniques have been developed for an off-road mobility model as an alternative to existing computational models to address the issue of high computational costs.**

Future Work



In order to further investigate the adaptive MOR capabilities developed in this study, the next step is to extend the approach to full-scale off-road vehicle mobility simulations, including **rovers** and ground vehicles. Furthermore, additional parameters, including vehicle maneuvering scenarios on different types of soil, will be considered in order to conduct more extensive performance evaluation of the **adaptive off-road mobility ROM**.

Acknowledgement

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